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AUTOGUIDANCE VIDEO SENSOR FOR DOCKING

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The Automated Rendezvous and Docking system is composed of two parts. The first part is the sensor which consists of a video camera ringed with two wavelengths of laser diode. The second part is a standard Remote Manipulator System(RMS) target used on the Orbiter that has been modified with three circular pieces of retro-reflective tape covered by optical filters which correspond to one of the wavelengths of laser diode. The sensor is on the chase vehicle and the target is on the target vehicle. The ARAD system works by pulsing one wavelength laser diodes and taking a picture. Then the second wavelength laser diodes are pulsed and a second picture is taken. One picture is subtracted from the other and the resultant picture is thresholded. All adjacent pixels above threshold are blobbed together(X and Y centroids calculated). All blob centroids are checked to recognize the target out of noise. Then the three target spots are windowed and tracked. The three target spot centroids are used to evaluate the roll, yaw, pitch, range, azimuth, and elevation. From that a guidance routine can guide the chase vehicle to dock with the target vehicle with the correct orientation.

BACKGROUND INTRODUCTION

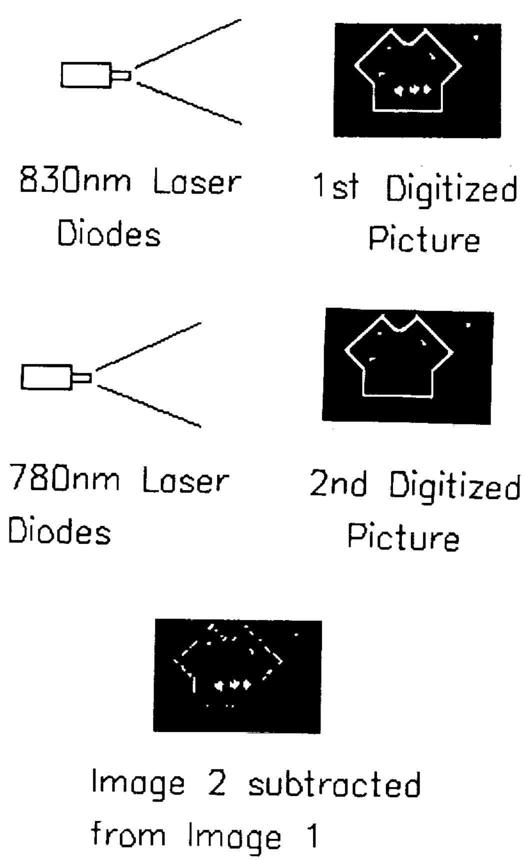
Past efforts in the rendezvous and docking area have been directed towards remotely piloted man-in-the-loop systems(OMV) or man directed systems (Orbiter). These video or visual based systems require a very large data stream to the ground operator or a man located on the rendezvous vehicle. To efficiently operate unmanned vehicles, minimize risk for hazardous or remote operations and reduce workload, automated rendezvous and docking (ARAD) techniques should be developed that will have application to the Cargo Transfer Vehicle(CTV), the Mars/Lunar Mission Vehicles and potentially the Shuttle Orbiter.

Recently, docking systems have been under investigation which use transponders and reflectors on the target vehicle and laser, radar and various computer based sensors have been used on the chaser vehicle for on-board range, range rate, and attitude determinations to support the ARAD function. Much work in this field has reached the level of real time system simulations and sensor testing at various NASA centers including the Marshall Space Flight Center(MSFC) and Johnson Space Flight Center(JSC). To define, develop, test, and evaluate various ARAD systems, a coordinated NASA effort is required. This effort will produce an implementation of an ARAD system that will have application to the Cargo Transfer Vehicle(CTV), Lunar/Mars Mission vehicles, and remote satellite servicing.

SYSTEM DESCRIPTION

There are five major components in any docking autoguidance system. They are: the autopilot, the control system, the docking latches, the sensor, and the docking target. The autopilot generates vehicle commands from the autoguidance sensor outputs. The vehicle control system executes the commands through control system thrusters for a spacecraft or joint motors of a robot arm. The docking latches could be a three point docking mechanism or a more complex mechanism to lock the chase and target vehicles together. This report concentrates only on the autoguidance video sensor development using simple retro-reflectors for a target. This report is divided into two sections: the present system and new developments.

The video docking sensor concept consists of five main components: laser illuminators, retro-reflective targets, a video camera, a frame grabber board, and a microprocessor. The laser illuminators are wide-angle laser diodes at two different wavelengths, 780nm and 830nm. The diodes are arranged such that they cover a 30-by-30 degree field-of-view. The target is composed of three circular pieces of retro-reflective tape each covered by 830nm filters that are mounted on an RMS target in a line with the center reflector on the center pole and the two outer reflectors equally spaced from the pole. This configuration is much more sensitive for yaw and pitch at zero degrees than a four corner flat target would be. The target is only 14.5 inches wide and the pole is four inches high. The sensor takes a picture with the 830nm laser diodes on and then the sensor takes another picture with the 780nm laser diodes on. The second picture is subtracted from the first and then a threshold is subtracted from the resultant image to give the reflector positions along with some noise(see figure 1: Range, azimuth, and elevation can be converted to X, Y, and Z ranges).



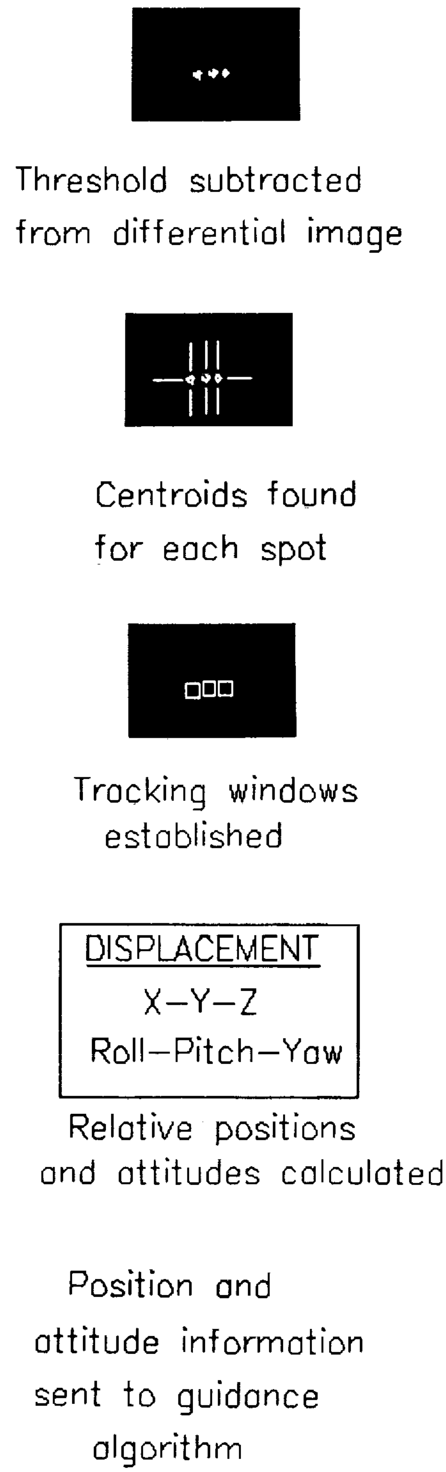


Figure 1: Steps to the Overall System Operation.

The software consists of two parts: initial target acquisition and reflector tracking.

The initial acquisition loop has three parts to it and they are: erosion and dilation, picking out bright spots, and identifying target reflectors out of noise spots.

Erosion and dilation are simply zeroing out the edge of all bright spots and then recreating the edge on all remaining bright spots. This elliminates all single point noise spots. Also, there is an edge created by pipes on target satellites which this elliminates along with any other edge effect. The target spots can not be too small or they will disappear.

There are two steps to picking out the bright spots and they are: grouping all adjacent pixels into spots, and calculating the X and Y centroids of each spot from the pixel data. The sensor scans the pixel data from left to right, top to bottom keeping track of the X coordinate(horizontal) and the Y coordinate(vertical) positions at all times. When a pixel is found with the intensity above the threshold, all adjacent pixels are checked to see if they are above the threshold. Then adjacencies to that layer of pixels are checked. All pixels above the threshold are zeroed out in the image as they are found so that they are not counted more than once. Adjacencies are checked for layer of pixels after layer of pixels until the spot is completely zeroed out of the image and stored with the X and Y coordinates. Then the scan continues where the first pixel for the spot was found. The data, X coordinates, and Y coordinates are then used to find the X and Y centroids of each spot.

$$\text{X\_Centroid} = \text{X\_Factor} / \text{Data\_Factor}$$

$$\text{Y\_Centroid} = \text{Y\_Factor} / \text{Data\_Factor}$$

$$\text{X\_Factor} = (\text{Data\_1} * \text{X\_Coordinate\_1}) + (\text{Data\_2} * \text{X\_Coordinate\_2}) + \ldots (\text{Data\_n} * \text{X\_Coordinate\_n})$$

$$\text{Y\_Factor} = (\text{Data\_1} * \text{Y\_Coordinate\_1}) + (\text{Data\_2} * \text{Y\_Coordinate\_2}) + \ldots (\text{Data\_n} * \text{Y\_Coordinate\_n})$$

$$\text{Data\_Factor} = \text{Data\_1} + \text{Data\_2} + \ldots \text{Data\_n}$$

After the X and Y centroids for all the spots are stored, the three target reflectors need to be identified from the noise spots. There has to be a minimum of three spots to even start to recognize the target. If there are three or more spots, these spot locations have to correspond to possible yaw and pitch target configurations to be identified as the reflectors. There is a triple nested loop for this section of code which examines three spots at any one time (any possible three spots is called a triplet). To reduce processing time, only non-redundant triplets are examined.

There are several steps inside the innermost loop needed to see if a triplet is the recognized reflector target. The first step is to calculate the length squared of each side of a triangle where the corners correspond to the triplet X and Y centroid locations. The second step is to find the base length or maximum length among the three lengths, which is the possible pixel length between the two outer reflectors. If there are two lengths of the same magnitude with one length shorter than the first two or if all three lengths are the same magnitude then there is no base length and the triplet is not the reflector target. The third step is to calculate the angle of the base length with respect to the horizontal(roll angle of possible reflector target). The fourth step is to use the roll angle to calculate an X and Y coordinate axis shift so that no matter how the target is rolled, accurate yaw and pitch target criteria can be used. These are the axis tranformation equations:

$$\text{Roll} = \text{Atan}\left(\frac{(\text{First\_Y\_Centroid} - \text{Third\_Y\_Centroid})}{(\text{Third\_X\_Centroid} - \text{First\_X\_Centroid})}\right)$$

Except 90 degrees. Assume centroid are sorted.

$$\text{Center\_X} = 0.5 * (\text{Outer\_Left\_Spot\_X\_Centroid} + \text{Outer\_Right\_Spot\_X\_Centroid})$$

$$\text{Center\_Y} = 0.5 * (\text{Outer\_Left\_Spot\_Y\_Centroid} + \text{Outer\_Right\_Spot\_Y\_Centroid})$$

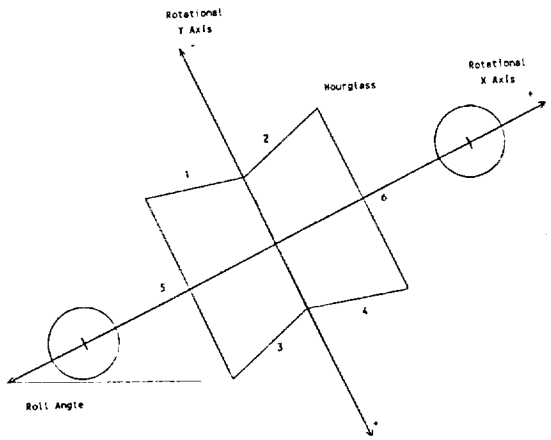
$$\text{Translational\_X\_Shift} = \text{Center\_X\_Spot\_Centroid} - \text{Center\_X}$$

$$\text{Translational\_Y\_Shift} = \text{Center\_Y\_Spot\_Centroid} - \text{Center\_Y}$$

$$\text{Rotational\_X\_Shift} = (\text{Translational\_X\_Shift} * \text{Cos}(\text{Roll})) - (\text{Translational\_Y\_Shift} * \text{Sin}(\text{Roll}))$$

$$\text{Rotational\_Y\_Shift} = (\text{Translational\_X\_Shift} * \text{Sin}(\text{Roll})) + (\text{Translational\_Y\_Shift} * \text{Cos}(\text{Roll}))$$

The reason for the axis tranformation is simple. There is a region centered between the two outer spots that is shaped like an hourglass with curved sides. The curved sides are approximated by four linear equations (see figure 2). This region corresponds to where the center spot would be if it were within 40 degees in yaw and pitch for the triplet to be the reflector target. The transformations are designed for the left-to-right increasing X coordinates and top-to-bottom increasing Y coordinates. Range is used as a target determining test for a reacquire. The final test is to compare the number of pixels in each spot to each other and if the numbers are close then accept the triplet as the target.




```
(1)  Y = ( 0.238*X )-( Y_Intercept*Pixel_Length_Of_Target )
(2)  Y = ( -0.238*X )-( Y_Intercept*Pixel_Length_Of_Target )
(3)  Y = ( -0.238*X )+( Y_Intercept*Pixel_Length_Of_Target )
(4)  Y = ( 0.238*X )+( Y_Intercept*Pixel_Length_Of_Target )
(5)  X = -X_Intercept*Pixel_Length_Of_Target
(6)  X = X_Intercept*Pixel_Length_Of_Target

X_Intercept = 0.22375 for small target
Y_Intercept = 0.17141 for small target
```

Figure 2: Hourglass Shaped Region Between Outer Spots.

Reflector tracking is done at two times per second using one frame grabber. First a frame is taken with the 830nm laser diodes active. Next, a frame is taken with the 780nm laser diodes active. The frame grabber subtracts the 780nm frame from the 830nm frame. Then the computer draws windows around the three reflectors and calculates the centroids of the windows. Range is used to calculate window size and window threshold(only pixel intensities above the threshold are included in the X and Y centroid calculations). The tracking itself (where to place next windows) is done by subtracting the the present X and Y centroids from the previous X and Y centroids to get the distances that are added to the present X and Y centroids to predict where the next windows should be. This works well for a constant rotational or translational velocity. At long range any chase vehicle yaw or pitch can shift the field-of-view enough for the reflectors to slip out of the windows because of the change in velocity. The same is true for close range translations. To counter this, translational and rotational accelerations should be sent from the control system Kalman filter back to the sensor. Then the X and Y centroids are used to calculate the roll, yaw, pitch, range, azimuth, and elevation (see appendix) which are sent to the control system.

The method to calculate the orientation information in the appendix is to first calculate roll using the equation from the acquisition section of this report. Then calculate Max_ Length between the two outer reflectors. Next, calculate the Rotational_(X and Y)_Shifts using equations from the acquisition section of this report. After that, use that information to calculate the yaw and pitch simultaneously. Finally, use the yaw to calculate range, azimuth, and elevation. Yaw, pitch, range, azimuth, and elevation all need to be calibrated for each lens used in any video autoguidance system. Calibrated yaw should be used to calculate pitch, range, azimuth, and elevation. Calibrated range should be used to calculate azimuth and elevation.

PERFORMANCE/STATUS

The present system works fairly well within twelve meters of the docking target. With the new geometry algorithm(see appendix) the range accuracy has been calibrated to one percent or below. The yaw, pitch, azimuth, and elevation have been calibrated to below one degree in accuracy. Dynamically, the system performs docking maneuvers with a reasonable degree of reliability using the air-bearing vehicle on the MSFC flatfloor facility, bringing the three point docking mechanism into latch position smoothly. Continuing testing with the Dynamic Overhead Target Simulator duplicating the full dynamic motion of a free flyer(like CTV) showed good correlation of sensor outputs and realtime relative positions. The autoguidance video sensor will be used to measure and guide the Space Station Freedom common berthing mechanism test article during dynamic berthing tests and to control the latching sequence.

NEW DEVELOPMENTS/ENHANCEMENTS

The current sensor system has some limitations and its performance can be enhanced through some new hardware improvements, some of which are now commercially off the shelf. Other changes will require some development along with continued testing and integration with more of the full system.

To increase the range of the system, two targets may be used. The large target would be three corner cube reflectors or three circular pieces of microprism. Both of these reflector

types return a much larger signal to the camera and the larger the target the more the resolution at range. Corner cubes have been used at 40 meters away. Corner cubes have one problem and that is, a limited target yaw or pitch angle before the spot size is significantly reduced. Microprism may overcome this problem. The small target would be the present target used now for close range because a large target image would grow too large for the camera. Using a two target system could help determine roll ambiguity because the offset between the two targets is known. Otherwise one of the outer reflectors would have to be larger than the other to distinguish between the two.

Another target concept is the optical collector. The entire target could be made into an optical collector that brings all the light to a central point where optical fibers send the light back out to the three light output positions(where the reflectors were). All three points would have the same intensity no matter how close the target is to the camera. There are no glass filters to cause any starring (glass reflects both wavelengths at just the right incident angle causing spot loss in the differenced image). The intensity would be far brighter because all the light on the entire target is being sent back at three points instead of just the light hitting the reflectors which means that this target could be seen from a much farther range.

In order to significantly improve the speed of the sensor system to fifteen readouts per second, two frame grabbers that are pre-programmable are required. In such a system, the two frame grabbers would have the same program loaded in before the docking run by the 386-based computer. One frame grabber grabs a frame with the 830nm laser diodes active. Then it grabs a frame with the 780nm laser diodes active. Next, it subtracts the 780nm frame from the 830nm frame(only in the three reflector tracking windows). Finally, it centroids the windows, sends the centroids to the computer, and receives the next window locations from the computer. The other frame grabber follows the same program steps. While one frame grabber is grabbing two frames the other frame grabber is performing the subtraction, centroiding, and window updates. The computer takes the centroids and calculates the yaw, roll, pitch, range, azimuth, and elevation and sends this information to the guidance system. The computer also controls the laser diodes and calculates the windows to send to the frame grabbers.

Improving the camera optics can improve the performance of the system. One improvement would be to control the integration time of the camera CCD. In sunlight the CCD may be saturated in the brightest parts(noise spots and reflectors). In some instants the noise from sunlight shining on mylar may be much brighter than the reflectors. A threshold above the reflector brightness may be set up as well as a threshold below the reflector brightness. But if the camera is saturated, this information is lost.

Another way of improving the optics is a concept put forth by a company called TRW. In this concept there are two cameras one with an 830nm optical filter covering and one with a 780nm optical filter covering. At the entrance to the sensor there is a lens and a beam spliter. Ringing the lens are the output optical fibers. Each input to these fibers is a laser diode (830nm or 780nm). In this concept both wavelengths of laser diode are active at the same time. The returned signal goes to both cameras through the beam spliter at the same time(both cameras and frame grabber are synchronized to the same clock). The frame from the 780nm camera is subtracted from the frame from the 830nm camera (only in the reflector readout windows). The remaining steps are just like the present system. In this way image differences that translate into noise on the resultant image are completely absent. If a camera malfunctions there is a backup. Both cameras can have a filter with a wide enough bandwidth to allow both wavelengths through that can be snapped into place where the tighter filter was if there is a camera malfunction. Then this system would operate just like the present system. If this system were implemented along with the two pre-programmable frame grabbers then the resultant system could possibly produce a thirty readout per second data rate.

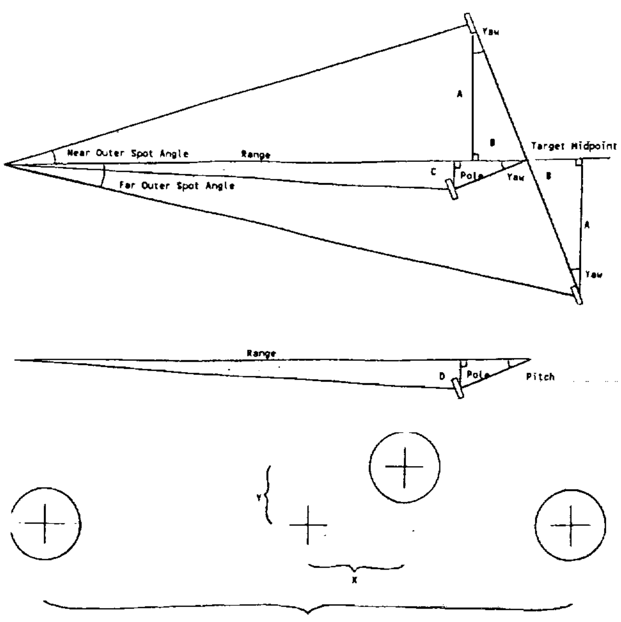
Another way to improve the sensor system is to have more than one camera each with a different lens and a different set of laser diodes. Each camera setup would be for a different range. At short range a wide field-of-view lens with laser diodes would be used. At longer range a narrow field-of-view lens with beam collimators to narrow the laser wide beam half power angle would be used with the laser diodes. The chase spacecraft would approach the target spacecraft with the longer range camera setups switching to the shorter range camera setups at the switchover ranges. This would greatly increase the range of the overall

system if implemented.

CONCLUSION

The current autoguidance video sensor, by acquiring and tracking three simple retroreflectors, provides accurate range and angular measurements for realtime docking of spacecraft. Any number of combinations of the above improvements may be implemented in the final system to create a highly reliable docking system with a reasonably good range from acquisition to the final docking. Other systems such as radar, laser rangers, GPS, etc will add autonomous rendezvous capability to the advanced docking system to provide fully Automated Rendezvous and Docking to support CTV, Mars/Moon missions and remote satellite servicing.

APPENDIX



$$\text{A}= 0.5*\text{Target\_Length}*\text{Cos}(\text{Yaw})$$
$$\text{B}= 0.5*\text{Target\_Length}*\text{Sin}(\text{Yaw})$$
$$\text{C}= \text{Pole\_Length}*\text{Sin}(\text{Yaw})*\text{Cos}(\text{Pitch})$$
$$\text{D}= \text{Pole\_Length}*\text{Sin}(\text{Pitch})$$

2*A corresponds to Max_Length

A+C corresponds to 0.5*Max_Length+Abs(X)

so

$$((\text{A}+\text{C})/(2*\text{A}))= (0.5*\text{Max\_Length}+\text{Abs}(\text{X}))/\text{Max\_Length}$$
$$\text{S}=(\text{Pole\_Length}/\text{Target\_Length})$$
$$\text{U}=(\text{Abs}(\text{X})/\text{Max\_Length})$$
$$0.5+ \text{S}*\text{Tan}(\text{Yaw})*\text{Cos}(\text{Pitch})= 0.5+\text{U}$$
$$\text{U}=(\text{S}*\text{Tan}(\text{Yaw})*\text{Cos}(\text{Pitch}))$$

and D corresponds to Abs(Y)

so

$$(\text{D}/(2*\text{A}))=(\text{Abs}(\text{Y})/\text{Max\_Length})$$
$$\text{V}=(\text{Abs}(\text{Y})/\text{Max\_Length})$$
$$\text{V}= \text{S}*\text{Sin}(\text{Pitch})/\text{Cos}(\text{Yaw})$$

$$\text{Sin}(\text{Pitch})=(1/\text{S})*\text{V}*\text{Cos}(\text{Yaw})$$
$$\text{Cos}(\text{Pitch})=(1/\text{S})*\text{U}/\text{Tan}(\text{Yaw})$$
$$\text{Sin}^2(\text{Pitch})=(1/\text{S})^2*\text{V}^2*\text{Cos}^2(\text{Yaw})$$
$$\text{Cos}^2(\text{Pitch})=(1/\text{S})^2*\text{U}^2/\text{Tan}^2(\text{Yaw})$$
$$\text{Sin}^2(\text{Pitch})+\text{Cos}^2(\text{Pitch})= 1$$
$$(1/\text{S})^2*(\text{V}^2*\text{Cos}^2(\text{Yaw})+\text{U}^2*\text{Cos}^2(\text{Yaw})/\text{Sin}^2(\text{Yaw}))= 1$$
$$\text{U}^2*\text{Cos}^2(\text{Yaw})+\text{V}^2*\text{Cos}^2(\text{Yaw})*\text{Sin}^2(\text{Yaw})= \text{S}^2*\text{Sin}^2(\text{Yaw})$$
$$\text{Sin}^2(\text{Yaw})= 0.5*(1-\text{Cos}(2*\text{Yaw}))$$
$$\text{Cos}^2(\text{Yaw})= 0.5*(1+\text{Cos}(2*\text{Yaw}))$$
$$0.5*\text{U}^2*(1+\text{Cos}(2*\text{Yaw}))+0.25*\text{V}^2*(1+\text{Cos}(2*\text{Yaw}))*(1-\text{Cos}(2*\text{Yaw}))= 0.5*\text{S}^2*(1-\text{Cos}(2*\text{Yaw}))$$
$$0.5*\text{U}^2+0.5*\text{U}^2*\text{Cos}(2*\text{Yaw})+0.25*\text{V}^2-0.25*\text{V}^2*\text{Cos}^2(2*\text{Yaw})= 0.5*\text{S}^2-0.5*\text{S}^2*\text{Cos}(2*\text{Yaw})$$
$$0.25*\text{V}^2*\text{Cos}^2(2*\text{Yaw})-0.5(\text{U}^2+\text{S}^2)*\text{Cos}(2*\text{Yaw})+(0.5*\text{S}^2-0.5*\text{U}^2-0.25*\text{V}^2)= 0$$
$$\text{G}= 0.25*\text{V}^2$$
$$\text{H}= -0.5*(\text{U}^2+\text{S}^2)$$
$$\text{I}=(0.5*\text{S}^2-0.5*\text{U}^2-0.25*\text{V}^2)$$
$$\text{Cos}(2*\text{Yaw})=(-\text{H}-(\text{H}^2-4*\text{G}*\text{I})^{0.5})/(2*\text{G})$$
$$\text{Calibrated\_Yaw}=\text{C1}*(\text{Yaw})+\text{C2}$$

where C1 is a slope and C2 is an angle offset (both determined by experimental data and linear regression)

Input Calibrated_Yaw into $\text{V}= \text{S}*\text{Sin}(\text{Pitch})/\text{Cos}(\text{Calibrated\_Yaw})$ to get Pitch.

$$\text{Calibrated\_Pitch}=\text{C3}*(\text{Pitch})+\text{C4}$$

where C3 is a slope and C4 is an angle offset (both determined by experimental data and linear regression)

for Y=0 (Pitch=0)

U= S*Tan(Yaw) then calibrate Yaw

for X=0 (Yaw=0)

V= S*Sin(Pitch) then calibrate Pitch

$$\text{Near\_Outer\_Spot\_Angle}= \text{Atan}(\text{A}/(\text{Range}-\text{B}))$$
$$\text{Far\_Outer\_Spot\_Angle}= \text{Atan}(\text{A}/(\text{Range}+\text{B}))$$
$$\text{Target\_Width\_Angle}= \text{Near\_Outer\_Spot\_Angle}+\text{Far\_Outer\_Spot\_Angle}$$
$$\text{Target\_Width\_Angle}= \text{Atan}(\text{A}/(\text{Range}-\text{B}))+\text{Atan}(\text{A}/(\text{Range}+\text{B}))$$
$$\text{Tan}(\text{Target\_Width\_Angle})= \frac{(\text{Tan}(\text{Near\_Outer\_Spot\_Angle})+\text{Tan}(\text{Far\_Outer\_Spot\_Angle}))}{(1-\text{Tan}(\text{Near\_Outer\_Spot\_Angle})*\text{Tan}(\text{Far\_Outer\_Spot\_Angle}))}$$
$$\text{Tan}(\text{Target\_Width\_Angle})= \frac{((\text{A}/(\text{Range}-\text{B}))+(\text{A}/(\text{Range}+\text{B})))}{(1-(\text{A}/(\text{Range}-\text{B}))*(\text{A}/(\text{Range}+\text{B})))}$$
$$\text{Tan}(\text{Target\_Width\_Angle})= 2*\text{A}*\text{Range}/(\text{Range}^2-\text{A}^2-\text{B}^2)$$
$$\text{Range}^2-(2*\text{A}/\text{Tan}(\text{Target\_Width\_Angle}))*\text{Range}-(\text{A}^2+\text{B}^2)= 0$$
$$\text{Target\_Width\_Angle}= 1.5338*10^{-3}*\text{Max\_Length}$$
$$\text{M}= -(2*\text{A}/\text{Tan}(\text{Target\_Width\_Angle}))$$
$$\text{N}= -(\text{A}^2+\text{B}^2)$$
$$\text{Range}= 0.5*(-\text{M}+(\text{M}^2-4*\text{N})^{0.5})$$
$$\text{Calibrated\_Range}=\text{Range}+\text{C5}$$

where C5 is the front focal length of the lens plus any other added length

$\text{Pix\_Shift}=651.975*\text{Atan}(\text{A}/(\text{Calibrated\_Range}-\text{B}))$ to shift image center to true target center

if Yaw greater than or equal to 0 (651.975 is pixels per radian)

$$\text{XA}=(\text{Pix\_Shift}/\text{Max\_Length})-0.5$$

if Yaw<0

$$\text{XA}= 0.5-(\text{Pix\_Shift}/\text{Max\_Length})$$
$$\text{X\_Shift}=\text{XA}*\text{Cos}(\text{Roll})$$
$$\text{Y\_Shift}= -\text{XA}*\text{Sin}(\text{Roll})$$
$$\text{Center\_X}= \text{X\_shift}+\text{Center\_X}$$
$$\text{Center\_Y}= \text{Y\_shift}+\text{Center\_Y}$$
$$\text{Azimuth}=1.5338*10^{-3}*(\text{Center\_X}-255.5)$$
$$\text{Elevation}=1.5338*10^{-3}*(191.6-\text{Center\_Y})$$
$$\text{Calibrated\_Azimuth}=\text{C6}*(\text{Azimuth})+\text{C7}$$

where C6 is a slope and C7 is an angle offset (both determined by experimental data and linear regression)

$$\text{Calibrated\_Elevation}=\text{C8}*(\text{Elevation})+\text{C9}$$

where C8 is a slope and C9 is an angle offset (both determined by experimental data and linear regression)